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- What is Two-Measure Theory (TMT)
- This work: The Two-Measure Standard Model (TMSM) and some of its main results.

Based on:

A. Kaganovich, ”Higgs inflation model with small non-minimal coupling constant”,
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A. Kaganovich, ”Two-Measure Electroweak Standard Model and Its Realization During Cosmological Evolution”,
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A. Kaganovich, ”The Two-Measure Theory and an Overview of Some of Its Manifestations”,
Universe 11 (2025) 11, 376

What is Two-Measure Theory (TMT)

<u>What are these two measures</u>

The structure of the original action in the Einstein's GR can be symbolically represented as

$$\boxed{\text{Riemannian geometry} \oplus \text{matter fields} = \text{Einstein's GR}}$$

In *the Palatini formulation of gravity* (P'GR), the metric tensor and the (affine) connection are treated as independent variables.
From the point of view of the applied mathematical apparatus, we inevitably come to the conclusion that in P'GR we are dealing with a 4-dimensional differentiable manifold (4D.Diff.Man.) at the stages of equipping it with an affine connection and a metric structure. That is, symbolically,

$$\boxed{\text{topological space} + \text{differential structure} = \text{4D.Diff.Man.}}$$

$$\boxed{(\text{4D.Diff.Man.} + \text{affine connection} + \text{metric}) + \text{matter fields} = \text{P'GR}}$$

What is Two-Measure Theory

What are these two measures

If we really believe in the effectiveness of mathematics in studying nature, we can go further and agree that *there must be a correspondence between the fundamental laws of nature and the structure of the mathematical apparatus necessary for their adequate description.*

Among the mathematical objects commonly used in field theories in 4D space-time, there is no the volume form, which can be defined as a 4-form on the 4D.Diff.Man. even before equipping it with an affine connection and a metric structure. The appropriate volume element can be constructed by means of 4 scalar functions φ_a ($a = 1, .., 4$) defined on the 4D.Diff.Man.

$$\begin{aligned} dV_\Upsilon &= \Upsilon d^4x \equiv \varepsilon^{\mu\nu\gamma\beta}\varepsilon_{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\gamma\varphi_c\partial_\beta\varphi_d d^4x \\ &= 4! d\varphi_1 \wedge d\varphi_2 \wedge d\varphi_3 \wedge d\varphi_4. \end{aligned}$$

What is Two-Measure Theory

After replacing Einstein's GR with Palatini's formulation of gravity, the construction of the Two-Measure theory can be seen as the next step in the realization of a clearly formulated idea: all possible degrees of freedom contained in the primordial action should be considered as independent of each other, and all relationships between them appear as a result of applying the principle of least action. Using dV_Υ as a volume measure in the action integral, we must consider the functions φ_a as degrees of freedom and, therefore, they are subject to variation when finding the extremum of the action.

The possibility of two alternative choices of the volume measure in the orientable manifold, $dV_g = \sqrt{-g}d^4x$ and $dV_\Upsilon = \Upsilon d^4x$, is based on the fact that under general coordinate transformations with a positive Jacobian (i.e. without changing the orientation of the manifold), the scalar densities $\sqrt{-g}$ and Υ are transformed according to the same rule. *The generally accepted approach in field theory is to use only the volume element dV_g, by default assuming that the volume element dV_Υ is redundant.*

What is Two-Measure Theory

A detailed arguments for ignoring a volume 4-form defined on a differentiable manifold are given by Robert Wald in his book "General Relativity" (in appendix B2).

Considering on n-dimensional manifold M the volume element $f(x)dx^1 \wedge ... \wedge dx^n$, with a non-vanishing function $f(x)$, Wald reasonably notes:
"if one is given only the structute of a manifold M,
there is no natural choice of volume element".

However, continuing Wald's analysis, we note that
the possibility of treating the volume element $\sqrt{|g|}dx^1 \wedge ... \wedge dx^n$ as natural exists only because in Einstein's GR the metric tensor is a dynamic variable, whose variation in the Einstein-Hilbert action leads to Einstein's equations.

What is Two-Measure Theory

Following Ward's approach, the original idea of the TMT might have been to include the volume element $f(x)dx^1 \wedge ... \wedge dx^n$ in the action integral, allowing $f(x)$ to be considered as another variable subject to variation in the action, along with the metric. But this modification of the theory leads to a trivial result: the function $f(x)$ plays the role of a Lagrange multiplier, and the Lagrangian term entering the action with the volume element $f(x)dx^1 \wedge ... \wedge dx^n$ turns out to be equal to zero after variation w.r. to f.

The idea of the TMT makes sense if we fully exploit the capabilities of the 4D.Diff.Man. Using the 4-form $dV_\Upsilon = 4! d\varphi_1 \wedge d\varphi_2 \wedge d\varphi_3 \wedge d\varphi_4$. and considering it as a volume element in the action integral together with $dV_g = \sqrt{-g}d^4x$, we, in fact, formulate new theory, where variation of the functions φ_i leads to an equation with a significant dynamic effect.

What is Two-Measure Theory

This is the essence of the TMT, and, returning to the analysis of Wald's argumentation, we see that **the volume elements** dV_g **and** dV_Υ **turn out to be equally natural**.

I believe that, within the framework of the formulated understanding of the significance of mathematical foundations, **there is no reason to continue to ignore the volume measure** dV_Υ **in field theory**. I would be very interested in discussing arguments against this fundamental claim.

TMT proposes using both volume measures as a linear combination. Thus, the novelty lies in the form of the Lagrangian density terms: instead of $\sqrt{-g}L_i$, TMT uses $(b_j\sqrt{-g} \pm \Upsilon)L_j$ $(j = 1, 2, ...n)$, where possible coefficients in front of Υ are extracted from the parentheses and absorbed by the corresponding redefinitions in L_j.
The possibility of two signs in front of Υ is explained by sign-indefinitness of Υ and plays a key role in achieving the TMT results.

What is Two-Measure Theory

Some technical notions. It is natural to begin with the gravitational term.

In general, the gravitational term of the action is

$$S_{gr} = \int \left[(\alpha \sqrt{-g} + \beta \Upsilon) \left(-\frac{M_P^2}{2} \right) g^{\mu\nu} R_{\mu\nu}(\Gamma) \right] d^4x. \qquad (1)$$

The factor α can be extract and absorbed by redifinition of M_P

After this, the factor $\frac{\beta}{\alpha}$ is absorbed by the appropriate redifinition of the functions φ_a from which Υ is built.

Then the gravitational term reduces to

$$S_{gr} = \int \left[(\sqrt{-g} + \Upsilon) \left(-\frac{M_P^2}{2} \right) g^{\mu\nu} R_{\mu\nu}(\Gamma) \right] d^4x. \qquad (2)$$

On conventional Higgs inflation models

One possible way to test what lies beyond the Standard Model as energy increases is to use cosmology.

Attempts to extend the application of the Higgs field condensate to energies on the scale of inflation ($\sim 10^{16}\ GeV$) lead to the idea that the inflaton is the Higgs field itself.

In conventional Higgs inflation models (F. Bezrukov, M. Shaposhnikov, A. Barvinsky, A. Kamenshchik, A. Starobinsky, J. Rubio, F. Bauer, D. Demir, T. Tenkanen, I. Gialamas, E. Tomberg, A. Shkerin, J. Ellis, I. Antoniadis, S. Zell, ...), agreement between the predicted inflation parameters and the observed CMB data is possible only if the non-minimal coupling constant ξ is huge: it varies in the range $10^4 \lesssim \xi \lesssim 10^8$ depending on the type of model, metric or Palatini.

This can be seen as a serious drawback of the models.

On conventional Higgs inflation models

In addition, conventional (electroweak SM) Higgs inflation models at the tree level do not provide a reasonable connection between physics at the SM energies and at the inflation energy scale.

In both the metric Higgs inflation models and the Palatini formulation, the transition to the Einstein frame is performed in the original action, which leads to a substantially non-polynomial Lagrangian. This is the cause of non-renormalizability, making it impossible to apply the RG equation method.

This extremely serious drawback forced the authors of the model to abandon further direct attacks and seek solutions to the problems by modifying the very foundations of the model.

Gravity + the SM bosonic sector in TMSM

Higgs Inflation with Small Non-Minimal Coupling Constant

The primordial action describing gravity and the bosonic sector of TMSM is chosen as follows:

$$S_{gr,bos} = S_{gr} + S_{nonmin} + S_H + S_g + S_{vac},$$

where

$$S_{gr} = -\frac{M_P^2}{2}\int d^4x\left(\sqrt{-g}+\Upsilon\right)R(\Gamma,g),$$

is the TMT-modification of the gravitational action in the Palatini formulation. M_P is the reduced Planck mass; Γ stands for affine connection; $R(\Gamma,g) = g^{\mu\nu}R_{\mu\nu}(\Gamma)$, $R_{\mu\nu}(\Gamma) = R^{\lambda}_{\mu\nu\lambda}(\Gamma)$ and $R^{\lambda}_{\mu\nu\sigma}(\Gamma) \equiv \Gamma^{\lambda}_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\gamma\sigma}\Gamma^{\gamma}_{\mu\nu} - (\nu \leftrightarrow \sigma)$.

Gravity + the SM bosonic sector in TMSM

The TMT-modification of the nonminimal coupling of the Higgs isodoublet

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

to the scalar curvature is described by

$$S_{nonmin} = -\xi \int d^4x \left(\sqrt{-g} + \Upsilon\right) R(\Gamma, g)|H|^2,$$

where $|H|^2 = H^\dagger H$. In our notations, in a theory with only the volume element $\sqrt{-g}d^4x$, the model of massless scalar field non-minimally coupled to scalar curvature would be conformally invariant if the parameter ξ were equal to $\xi = -\frac{1}{6}$.

Gravity + the SM bosonic sector in TMSM

The TMT- primordial action S_H for the Higgs field H has a standard $SU(2) \times U(1)$ gauge invariant structure

$$
\begin{aligned}
S_H &= \int d^4x \Big[\Big(b_k \sqrt{-g} - \Upsilon \Big) g^{\alpha\beta} \left(D_\alpha H \right)^\dagger D_\beta H \\
&- \Big(b_p \sqrt{-g} + \Upsilon \Big) \lambda |H|^4 - \Big(b_p \sqrt{-g} - \Upsilon \Big) m^2 |H|^2 \Big],
\end{aligned}
$$

where additional factors associated with the existence of two types of volume elements are selected in a special way.

Υ may be preceded by either "$+$" or "$-$", since Υ is, in general, sign-indefinite. This feature of TMT and this choice of signs make it possible to realize the idea of Higgs inflation for small ξ. The parameters $b_k > 0$, $b_p > 0$, $\lambda > 0$ and $m^2 > 0$ are chosen to be positive.

Gravity + the SM bosonic sector in TMSM

The standard definition for the operator

$$D_\mu = \partial_\mu - ig\hat{\mathbf{T}}\mathbf{A}_\mu - i\frac{g'}{2}\hat{Y}B_\mu,$$

is used with the primordial gauge coupling parameters g and g'. Here, as usually, $\hat{\mathbf{T}}$ stands for the three generators of the $SU(2)$ group and $\hat{Y}$ is the generator of the $U(1)$ group.

With standard notations $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + g\mathbf{A}_\mu \times \mathbf{A}_\nu$ and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ for the field strengths of the isovector $\mathbf{A}_\mu$ and isoscalar B_μ, the gauge fields primordial action has the standard $SU(2) \times U(1)$ gauge invariant structure

$$S_g = \int d^4x\left(b_k\sqrt{-g} - \Upsilon\right)\left[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\mathbf{F}_{\mu\nu}\mathbf{F}_{\alpha\beta} - \frac{1}{4}g^{\mu\alpha}g^{\nu\beta}B_{\mu\nu}B_{\alpha\beta}\right],$$

where the density of the volume measure is chosen to be the same as in the kinetic term of S_H.

Gravity + the SM bosonic sector in TMSM

The presence of two volume measures in TMT is the reason for possibility of two vacuum-like terms in the primordial action. They are defined by

$$S_{vac} = \int d^4x \left(-\sqrt{-g}V_1 - \frac{\Upsilon^2}{\sqrt{-g}}V_2 \right).$$

We use a parametrization $|V_2| = (qM_P)^4$ and the choice

$$V_1 = V_2 = -(qM_P)^4 \approx -(10^{16} GeV)^4,$$

which means that $q^4 \approx 3 \cdot 10^{-10}$.

The reason for adding such a term with V_2 : The scalar $\zeta = \frac{dV_g}{dV_\Upsilon} = \frac{\Upsilon}{\sqrt{-g}}$ has a key role in all TMT models. The term $\propto \Upsilon = \zeta(x)\sqrt{-g}$, which can be expected as the contribution of quantum-gravitational effects to a vacuum-like action, does not contribute to the eqs. of motion. Therefore, the next in powers of ζ vacuum-like term can be of the form $\propto \zeta^2\sqrt{-g} = \Upsilon^2/\sqrt{-g}$.

Gravity + the SM bosonic sector in TMSM

If a homogeneous and isotropic spatially flat universe is described by the Friedmann-Robertson-Walker (FRW) metric $ds^2 = dt^2 - a^2(t)d\vec{x}^{\,2}$, then the value of the classical scalar field $\phi(t)$ which drives the cosmological evolution at the moment t of the cosmological time, is a result of *the cosmological averaging of an inhomogeneous field* $\phi(x)$. Applying this idea to the Higgs isodoublet $H(x)$ we have

$$\langle H(x)\rangle_{cosm.\,aver.} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\phi(t) \end{pmatrix} \tag{3}$$

where the field $\phi(t)$ is the only nonzero component of the Higgs isodoublet $H(x)$ obtained as a result of the cosmological averaging. As usually in Higgs inflation models, $\phi(t)$ plays the role of the inflaton field.

Gravity + the SM bosonic sector in TMSM

A specific *dynamic* feature of TMT is that **the ratio of volume measures**

$$\zeta = \frac{dV_g}{dV_\Upsilon} = \frac{\Upsilon}{\sqrt{-g}}$$

appears in all equations of motion, and all coupling constants become ϕ-dependent due to $\zeta(\phi)$.

On the other hand, **a self-consistency condition of the equations obtained by varying the metric $g_{\mu\nu}$ and the functions φ_a, determines ζ as the rational function of ϕ,** which we call **a constraint**.

Although Einstein's equations are reproduced exactly, due to the constraint the TMT-effective energy-momentum tensor in general has a structure fundamentally different from conventional models; moreover, the function $\zeta(\phi)$ has a significant influence on the dynamics of matter fields. Thus, it turns out that **the function $\zeta(\phi)$ plays a key role in obtaining all the new results.**

Gravity + the SM bosonic sector in TMSM

The structure of TMT is such that the original action must include gravity, that is *TMT models cannot be formulated directly in Minkowski space*. The equations derived from the principle of least action describe a self-consistent system of gravity and matter.

Represented in Einstein frame, this system of equations is valid at any energy accessible to classical field theory.

Incorporating the electroweak SM into TMT in a way that fully accounts for the structure of TMT leads to a theory we call the Two-Measure SM (TMSM).

In the context of cosmology, **the TMSM describes what the tree level SM looks like at various stages of cosmological evolution, which is equivalent to the description at any admissible energy.**

Particular attention: in TMSM the description of matter in Minkowski space can be achieved only by the limiting transition from curved space-time, that is, by approaching the vacuum state with a zero cosmological constant.

Gravity + the SM Higgs field in TMSM

Following the prescription of the TMT procedure, we first obtain equations by varying the primordial action w.r. to φ_a, $\Gamma^{\lambda}_{\mu\nu}$, $g_{\mu\nu}$ and ϕ.

It is very importat that as a result of varying w.r. to φ_a an arbitrary integration constant $\mathcal{M}$ appears. In fact, representing the primordial action in the form $S_{gr,bos} = \int(\sqrt{-g}L_1 + \Upsilon L_2)d^4x$ and varying the scalar functions φ_a of which Υ is built we get $B^{\mu}_{a}\partial_{\mu}L_2 = 0$, where $B^{\mu}_{a} = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_{\nu}\varphi_b\partial_{\alpha}\varphi_c\partial_{\beta}\varphi_d$. Since $Det(B^{\mu}_{a}) \propto \Upsilon^3$ it follows that if

$$\text{everywhere} \quad \Upsilon(x) \neq 0,$$

the equality $L_2 = \mathcal{M}$ must be satisfied, where $\mathcal{M}$ is a constant of integration with the dimension of $(mass)^4$.

Gravity + the SM Higgs field in TMSM

Thus, solution exists only if $\Upsilon(x) > 0$ or only if $\Upsilon(x) < 0$.
Since eq. $B^{\mu}_{a}\partial_{\mu}L_2 = 0$ is one of the equations resulting from the principle of least action, *only those solutions of the system of equations are valid for which the corresponding* $\Upsilon(x)$ *is sign-definite*,
Since the original metric $g_{\mu\nu}$ is regular, that is $g = \det(g_{\mu\nu}) < 0$, the validity of the solutions regarding the fulfillment of the condition on the sign of Υ can be controlled by checking the sign of the scalar $\zeta = \frac{\Upsilon}{\sqrt{-g}}$.
We choose the integration constant $\mathcal{M}$ so that the TMT-effective vacuum energy density Λ satisfies the natural constraint $0 \leq \Lambda \lesssim (\textit{electroweak scale})^4$. It is achieved if

$$\mathcal{M} = \pm 2(qM_P)^4(1+\delta), \qquad \textit{where} \qquad |\delta| \sim \left(\frac{v}{M_P}\right)^4 \ll 1.$$

Gravity + the SM Higgs field in TMSM

Two possible signs of $\mathcal{M}$ correspond to two possible signs of ζ in the vacuum: $\zeta_v = \pm\sqrt{V_1/V_2} = \pm 1$.
Choosing the solution with $\mathcal{M} = +2(qM_P)^4$ we get
$\zeta_v = \sqrt{V_1/V_2} = 1$.
Thus, the sign of $\Upsilon(x)$ is fixed only after the choice of a solution, and only those cosmological solutions (together with their initial conditions) are valid for which $\zeta(x)$ is positive throughout the evolution of the universe. In other words, these solutions lose their validity when we try to extend them to the region where ζ crosses zero and becomes negative.
The condition $\Upsilon(x) > 0$ is equivalent to the sign-definitness of the volume 4-form $\Upsilon(x)d^4x$ on the 4D-space-time manifold. From the geometrical point of view this means that the differentiable manifold of our Universe acquires a certain orientation. This is actually **a new TMT effect consisting of a spontaneous restoration of the orientation of the space-time manifold**.

Gravity + the SM Higgs field in TMSM

The equations of motion are then rewritten in the Einstein frame using the Weyl transformation

$$\tilde{g}_{\mu\nu} = (1+\zeta)\Omega g_{\mu\nu},$$

where $\Omega = 1 + \xi \frac{\phi^2}{M_P^2}$.

The self-consistency condition of the resulting system of equations determines ζ as the following function of ϕ and $X_\phi = \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$:

$$\zeta(\phi, X_\phi) =$$

$$\frac{2(qM_P)^4\zeta_v(1+\zeta_v) - (2b_p-1)\frac{\lambda}{4}\phi^4 - (2b_p+1)\frac{m^2}{2}\phi^2 + (1+b_k)\Omega X_\phi}{2(qM_P)^4(1+\zeta_v) + \frac{\lambda}{4}\phi^4 - \frac{m^2}{2}\phi^2 - (1+b_k)\Omega X_\phi}$$

Important:

When using Palatini gravity, the function ζ does not have its own differential equation of motion and is determined only by the constraint.

Gravity + the SM Higgs field in the cosmological realization of TMSM

The grav.eqs. in the Einstein frame have the canonical GR form

$$R_{\mu\nu}(\tilde{g}) - \frac{1}{2}\tilde{g}_{\mu\nu}R(\tilde{g}) = \frac{1}{M_P^2}T_{\mu\nu}^{(eff)}.$$

Here $R_{\mu\nu}(\tilde{g})$ and $R(\tilde{g})$ are the Ricci tensor and the scalar curvature of the metric $\tilde{g}_{\mu\nu}$, respectively.
The ζ-dependent form of the TMT-effective energy-momentum tensor $T_{\mu\nu}^{(eff)}$ is

$$T_{\mu\nu}^{(eff)}(\phi;\zeta) = \frac{1}{\Omega}\left(\frac{b_k-\zeta}{1+\zeta}\phi_{,\mu}\phi_{,\nu} + \tilde{g}_{\mu\nu}X_\phi\right) + \tilde{g}_{\mu\nu}U_{eff}(\phi;\zeta),$$

where

$$U_{eff}(\phi;\zeta) = \frac{1}{(1+\zeta)^2\Omega^2}\left[q^4M_P^4(\zeta_v^2-\zeta^2)+(1-b_p)\frac{\lambda}{4}\phi^4-(1+b_p)\frac{m^2}{2}\phi^2\right].$$

After substituting $\zeta(\phi, X_\phi)$ into $T^{(eff)}_{\mu\nu}(\phi;\zeta)$ its expression reduces to

$$T^{(eff)}_{\mu\nu} =$$

$$= \frac{K_1(\phi)}{\Omega}\cdot\left(\phi_{,\mu}\phi_{,\nu}-\tilde{g}_{\mu\nu}X_\phi\right)-K_2(\phi)\cdot\frac{X_\phi}{M_P^4}\Big(\phi_{,\mu}\phi_{,\nu}-\frac{1}{2}\tilde{g}_{\mu\nu}X_\phi\Big)+\tilde{g}_{\mu\nu}U_{eff}(\phi),$$

where the TMT effective potential $U_{eff}(\phi)$ and functions $K_1(\phi)$ and $K_2(\phi)$ are defined as follows:

$$U_{eff}(\phi) = \frac{q^4\Big[(\zeta_v+b_p)\lambda\phi^4-2(\zeta_v-b_p)m^2\phi^2\Big]+\frac{1}{M_P^4}\left(\frac{\lambda}{4}\phi^4-\frac{m^2}{2}\phi^2\right)^2}{4\Omega^2\left[q^4(1+\zeta_v)^2+(1-b_p)\frac{\lambda}{4}\frac{\phi^4}{M_P^4}-(1+b_p)\frac{m^2\phi^2}{2M_P^4}\right]},$$

$$K_1(\phi) =$$

$$= \frac{2q^4(1+\zeta_v)(b_k-\zeta_v)+(2b_p+b_k-1)\frac{\lambda}{4}\frac{\phi^4}{M_P^4}+(2b_p+1-b_k)\frac{m^2\phi^2}{2M_P^4}}{2\left[q^4(1+\zeta_v)^2+(1-b_p)\frac{\lambda}{4}\frac{\phi^4}{M_P^4}-(1+b_p)\frac{m^2\phi^2}{2M_P^4}\right]},$$

$$K_2(\phi) = \frac{(1+b_k)^2}{2\left[q^4(1+\zeta_v)^2+(1-b_p)\frac{\lambda}{4}\frac{\phi^4}{M_P^4}-(1+b_p)\frac{m^2\phi^2}{2M_P^4}\right]}.$$

Gravity + the SM Higgs field in the cosmological realization of TMSM

Instead of the system of equations it is more convenient to proceed with the TMT-effective action, the variation of which gives all equations. As usual, if $\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} > 0$, the TMT-effective energy-momentum tensor $T_{\mu\nu}^{(eff)}$ can be rewritten in the form of a perfect fluid. Then the pressure density plays the role of the matter Lagrangian in the effective action, and we arrive at the *tree level TMT-effective action*

$$S_{eff}^{(tree)} = \int \left(-\frac{M_P^2}{2} R(\tilde{g}) + L_{eff}^{(tree)}(\phi, X_\phi) \right) \sqrt{-\tilde{g}} d^4 x,$$

with the following tree level TMT-effective Lagrangian

$$L_{eff}^{(tree)}(\phi, X_\phi)) = K_1(\phi) \frac{X_\phi}{\Omega} - \frac{1}{2} K_2(\phi) \frac{X_\phi^2}{M_P^4} - U_{eff}(\phi).$$

Note that $L_{eff}^{(tree)}(\phi, X_\phi))$ has the form typical for K-essence models.

Higgs Inflation with Small Non-Minimal Coupling Constant

To study inflation, as usual, it is convenient to redefine the field ϕ using the relation $\phi = \frac{M_P}{\sqrt{\xi}}\sinh\left(\sqrt{\xi}\frac{\varphi}{M_P}\right)$.
Then the TMT-effective potential $U_{eff}(\phi)$ is expressed in terms of φ

$$U_{eff}(\varphi) = \frac{\lambda M_P^4}{4\xi^2}\tanh^4 z \cdot F(z), \quad \text{where } z = \sqrt{\xi}\frac{\varphi}{M_P},$$

$$F(z) = \frac{(\zeta_v + b_p)q^4 + \frac{\lambda}{16\xi^2}\sinh^4 z - \frac{\lambda m^2}{4\xi M_P^2}\sinh^2 z + \frac{m^4}{4\lambda M_P^4} - (\zeta_v - b_p)\frac{2q^4\xi m^2}{\lambda M_P^2 \sinh^2 z}}{(1+\zeta_v)^2 q^4 + (1-b_p)\frac{\lambda}{4\xi^2}\sinh^4 z - (1+b_p)\frac{m^2}{2M_P^2\xi}\sinh^2 z}.$$

$U_{eff}(\varphi)$ has one plateau of height $U_{eff}|_{plateau} \approx \frac{\lambda}{8\xi^2} M_P^4$

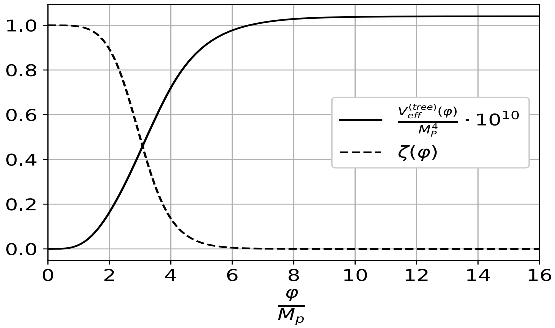

Figure: The solid curve is a plot of the TMT-effective potential $10^{10} U_{eff}(\varphi)/M_P^4$ with the parameters $q^4 = 3 \cdot 10^{-10}$, $\lambda = 2.3 \cdot 10^{-11}$, $\xi = \frac{1}{6}$, $b_p = \frac{1}{2}(1+\delta_p)$ $\delta_p = 10^{-8}$, $m = 0.7 GeV$. The dashed line is the graph of $\zeta(\varphi)$ defined by the constraint represented via φ. As it turns out, for $\varphi \gtrsim 6M_P$, the terms $\propto \frac{X_\varphi}{M_P^4}$ are extremely small compared to the other terms. The $\zeta(\varphi)$ curve intersects $\zeta = 0$ at $\varphi \approx 14M_P$; the intersection point is very sensitive to the choice of δ_p. The choice $\delta_p = 10^{-8}$ provides sufficient duration of the slow-roll inflation.

Higgs Inflation with Small Non-Minimal Coupling Constant

Applying the model to the very beginning of the classical evolution of the Universe showed (JCAP 03 (2026) 006) that due to the K-essence type structure of $L_{eff}^{(tree)}(\phi, X_\phi))$, under certain initial conditions, cosmological dynamics can begin with a ”pathological” and even phantom regime preceding inflation.

However, **if evolution begins with normal dynamics, then it proceeds only as inflation, and the problem of initial conditions for the onset of inflation does not arise.**

The K-essence type structure of $L_{eff}^{(tree)}(\phi, X_\phi))$ has very little effect on the inflation.

Higgs Inflation with Small Non-Minimal Coupling Constant

In slow-roll approximation $\frac{1}{2}\dot{\varphi}^2 \ll U_{eff}$ and $|\ddot{\varphi}| \ll |U_{eff}^{(tree)\,\prime}| \approx 3H|\dot{\varphi}|$ cosmological equations reduce to

$$3\frac{\dot{a}^2}{a^2}\dot{\varphi} \simeq U_{eff}^{(tree)\,\prime}(\varphi), \tag{4}$$

$$\frac{\dot{a}^2}{a^2} \simeq \frac{1}{3M_P^2}U_{eff}^{(tree)}(\varphi), \tag{5}$$

from where it follows that the flatness conditions imposed on $U_{eff}^{(tree)}(\varphi)$

$$\epsilon = \frac{M_P^2}{2}\left(\frac{U_{eff}^{(tree)\,\prime}}{U_{eff}^{(tree)}}\right)^2 \ll 1, \qquad |\eta| = M_P^2\left|\frac{U_{eff}^{(tree)\,\prime\prime}}{U_{eff}^{(tree)}}\right| \ll 1 \tag{6}$$

must be satisfied.

Higgs Inflation with Small Non-Minimal Coupling Constant

At this stage, our main goal is to understand how Higgs inflation with a small model parameter ξ becomes possible in TMSM. First of all, choosing the value of $\xi = \frac{1}{6}$ we must find the value of the primordial parameter λ in the TMT-effective potential in such a way that the model predictions agree with the CMB data.

Using the duration of inflation, measured by the number of e-folds N, we obtain

$$\epsilon \approx \frac{1}{8\xi N^2} \qquad \text{and} \qquad \eta \approx -\frac{1}{N}.$$

The power spectrum of scalar perturbations $\mathcal{A}_s$, measured in the CMB at large scales, can be expressed (Lyth, Riotto, 1999) in terms of the slow-roll parameter ϵ as $\mathcal{A}_s \approx \frac{1}{24\pi^2\epsilon}\frac{U_{eff}}{M_P^4}$, and for large N we get

$$\mathcal{A}_s \approx \frac{\lambda N^2}{24\pi^2\xi}.$$

Higgs Inflation with Small Non-Minimal Coupling Constant

With the Planck's observational data bound $\mathcal{A}_s \approx 2.1 \cdot 10^{-9}$ we get the relation

$$\lambda \approx 5 \cdot 10^{-7} \frac{\xi}{N^2}.$$

For $N = 60$, with $\xi = \frac{1}{6}$, we obtain

$$\lambda \approx 2.3 \cdot 10^{-11}.$$

For the scalar spectral index $n_s = 1 - 6\epsilon + 2\eta$ and the tensor-to-scalar ratio $r = 16\epsilon$ we find for $N = 60$

$$n_s = 1 - \frac{2}{N} - \frac{3}{4\xi N^2} \approx 0.965 \qquad \text{and} \qquad r = \frac{2}{\xi N^2} \approx 0.003,$$

which are in good agreement with the observational results of the Planck 2018 survey and the BICEP/Keck measurement.

Higgs Inflation with Small Non-Minimal Coupling Constant

Now turn to the key point: how the studied TMSM allows us to implement the Higgs inflation model with $\xi = \frac{1}{6}$ while satisfying the observed cosmological data due to $\lambda \sim 10^{-11}$, on the one hand, and to satisfy the requirement of $\lambda \sim 0.1$ near vacuum imposed by the SM, on the other hand.

Let's analyze the ϕ-equation in the ζ-dependent form (which also has a non-canonical kinetic term).

$$
\begin{aligned}
& \frac{1}{\sqrt{-\tilde{g}}}\partial_\mu\left(\frac{b_k-\zeta}{(1+\zeta)\Omega}\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right)+\frac{\xi}{\Omega^2}R(\tilde{g})\phi \\
+ \quad & \frac{b_p+\zeta}{(1+\zeta)^2\Omega^2}\lambda\phi^3+\frac{b_p-\zeta}{(1+\zeta)^2\Omega^2}m^2\phi=0,
\end{aligned}
$$

At a certain moment of cosmological evolution, the increasing function $\zeta(\varphi)$ crosses the value $b_p \approx \frac{1}{2}$; as a result, the mass term proportional to $(b_p-\zeta)m^2\phi$ changes sign to the "wrong" one, which provides SSB in the standard way of GWS theory.

Higgs Inflation with Small Non-Minimal Coupling Constant

Near vacuum, where $\zeta = \zeta_v = 1$, the TMT effective Lagrangian $L_{eff}^{(tree)}(\phi, X_\phi))$ can be represented in Minkowski apace

$$L_{eff}^{(tree)} = \frac{1}{2}(b_k - 1)\eta^{\alpha\beta}\,(D_\alpha H)^\dagger\, D_\beta H - \frac{1+b_p}{4}\lambda|H|^4 + \frac{1-b_p}{4}m^2|H|^2$$

To apply functional quantization, it must first be reduced to the canonical form of the GWS theory by the redefinition of the Higgs isodoublet $\sqrt{\frac{b_k-1}{2}} \cdot H = \mathcal{H}$ and, correspondingly, of the parameters: $\lambda_{SM} = \frac{1+b_p}{(b_k-1)^2}\lambda, \quad m^2_{SM} = \frac{1-b_p}{2(b_k-1)}m^2$

Then kinetic term becomes canonical and the potential reduces to

$$V(\mathcal{H}) = \lambda_{SM}|\mathcal{H}|^4 - m^2_{SM}|\mathcal{H}|^2$$

Higgs Inflation with Small Non-Minimal Coupling Constant

Then the VEV with $v = m_{SM}/\sqrt{\lambda_{SM}} \approx 246\,GeV$ and the Higgs boson mass $m_h \approx 125\,GeV$, as in the GWS theory, are obtained if

$$b_k - 1 = 1.6 \cdot 10^{-5} \quad \text{and} \quad m \approx 0.7\,GeV,$$

In the vacuum, the smallness of $b_k - \zeta_v = b_k - 1$ compensates for the smallness of λ:

$$\lambda_{SM} = \frac{1 + b_p}{(b_k - 1)^2}\lambda$$

The change of λ during cosmological evolution from $\lambda \sim 10^{-11}$ at the inflationary stage to $\lambda_{SM} \sim 0.1$ near vacuum is one of the results of "the classical running TMT-effective coupling constants".

The Two-Measure Standard Model (TMSM). Main idea

The field $\phi(t)$ together with the function ζ and the curvature describe the classical cosmological background on which the SM exists as a local quantum field theory.

If we consider the GWS theory as a pattern, then at each specific stage of the evolution of the cosmological background, the particle field theory is realized as a cosmologically modified copy of the SM.

The realization of this idea is analytically possible if the function ζ is nearly constant. This occurs at two stages of cosmological evolution:

- At $\phi \gtrsim 14.2M_P$ (which corresponds to $\varphi \gtrsim 6M_P$), i.e. during slow-roll inflation, ζ is bounded by $0 < \zeta \lesssim 5 \cdot 10^{-3}$.
- At $\phi \lesssim 0.1M_p$, where, neglecting relative corrections of the order $\lesssim 5 \cdot 10^{-7}$, one can use the value of ζ in the vacuum, $\zeta_v = 1$

The Two-Measure Standard Model (TMSM). Summary of the main results

A detailed study conducted in papers JCAP 03 (2026) 006 and Symmetry 18 (2026) 3, 508 shows that

- The GWS theory is reproduced when the Yukawa constant in the primordial action is chosen to be universal for three generations of fermions.
- During cosmological evolution, from inflation to a state close to vacuum, the classical ”running” parameters of the gauge and Yukawa couplings change by several orders of magnitude.

The Two-Measure Standard Model (TMSM). Summary of the main results

- On the slow-roll inflationary background, the appropriate copy of SM is renormalizable.
- Using classical-level results and due to renormalizability, it is shown that taking quantum corrections into account in the one-loop approximation does not violate vacuum stability during inflation. This is achieved due to the smallness of fermion contributions compared to boson contributions to the one-loop effective potential.
- Quantum corrections in the one-loop approximation preserves the slow-roll inflation regime.
- The fermion preheating model is described as a preliminary study of preheatig after inflation.

Bosonic fields near vacuum. $\zeta_{back} = \zeta_v = 1$

(partially presented already in slides 34, 35)

$$
\begin{aligned}
& L^{(bos)}_{(on\, back\, near\, vac)} = \\
= \quad & \frac{1}{2}(b_k - 1)\eta^{\alpha\beta}\left(D_\alpha H\right)^\dagger D_\beta H - \frac{1+b_p}{4}\lambda|H|^4 + \frac{1-b_p}{4}m^2|H|^2\Big] \\
- \quad & \frac{1}{4}(b_k - 1)\eta^{\mu\alpha}\eta^{\nu\beta}\left(\mathbf{F}_{\mu\nu}\mathbf{F}_{\alpha\beta} + B_{\mu\nu}B_{\alpha\beta}\right).
\end{aligned}
$$

To bring the Lagrangian to the canonical form of the electroweak GWS theory, we make the following redefinitions:

$\sqrt{\frac{b_k-1}{2}} \cdot H = \mathcal{H};\quad \lambda_{SM} = \frac{1+b_p}{(b_k-1)^2}\lambda,\quad m^2_{SM} = \frac{1-b_p}{2(b_k-1)}m^2$

$\sqrt{b_k - 1} \cdot \mathbf{A}_\mu = \mathcal{A}_\mu,\quad \sqrt{b_k - 1} \cdot B_\mu = \mathcal{B}_\mu,$

$\frac{g}{\sqrt{b_k-1}} = \mathrm{g},\quad \frac{g'}{\sqrt{b_k-1}} = \mathrm{g}';$

$\sqrt{b_k - 1} \cdot \mathbf{F}_{\mu\nu} = \mathcal{F}_{\mu\nu} = \frac{\partial \mathcal{A}_\nu}{\partial x^\mu} - \frac{\partial \mathcal{A}_\mu}{\partial x^\nu} + \mathrm{g}\mathcal{A}_\mu \times \mathcal{A}_\nu,$

$\sqrt{b_k - 1} \cdot B_{\mu\nu} = \mathrm{B}_{\mu\nu} = \frac{\partial \mathrm{B}_\nu}{\partial x^\mu} - \frac{\partial \mathrm{B}_\mu}{\partial x^\nu}$

Bosonic fields near vacuum

$$
\begin{aligned}
L^{(bos)}|_{(on\,back\,near\,vac)} &= \eta^{\alpha\beta}\left(\mathcal{D}_\alpha\mathcal{H}\right)^\dagger\mathcal{D}_\beta\mathcal{H} - V(\mathcal{H}) \\
&- \frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}\left(\mathcal{F}_{\mu\nu}\mathcal{F}_{\alpha\beta} + \mathcal{B}_{\mu\nu}\mathcal{B}_{\alpha\beta}\right),
\end{aligned}
$$

$$
V(\mathcal{H}) = \lambda_{SM}|\mathcal{H}|^4 - m_{SM}^2|\mathcal{H}|^2.
$$

$$V(\mathcal{H}) = \lambda_{SM}|\mathcal{H}|^4 - m_{SM}^2|\mathcal{H}|^2$$

$$\langle 0|\mathcal{H}|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix}$$

$$v^2 = \frac{m_{SM}^2}{\lambda_{SM}} = \frac{1-b_p}{1+b_p}\cdot\frac{(b_k-1)m^2}{2\lambda} = \frac{(b_k-1)m^2}{6\lambda}$$

$$\mathcal{H} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix}, \qquad \langle 0|h|0\rangle = 0$$

$$V(h) = \frac{1}{4}\lambda_{SM}\left(2vh + h^2\right)^2 - \frac{1}{4}\lambda_{SM}v^4$$

$$m_h^2 = 2\lambda_{SM}v^2 = \frac{m^2}{2(b_k-1)}$$

Gauge fields near vacuum

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(\mathcal{A}^1_{\mu} \mp i\mathcal{A}^2_{\mu}),$$

$$Z_{\mu} = \sin\theta_W\,\mathcal{B}_{\mu} - \cos\theta_W\,\mathcal{A}^3_{\mu}, \quad A_{\mu} = \cos\theta_W\,\mathcal{B}_{\mu} + \sin\theta_W\,\mathcal{A}^3_{\mu},$$

where the Weinberg angle θ_W is given by $\tan\theta_W = \frac{\mathrm{g}'}{\mathrm{g}} = \frac{g'}{g}$.
For generated by the Higgs phenomenon masses of W and Z-bosons the standard expressions are obtained
$M_W = \frac{\mathrm{g}}{2}\cdot v;\ \ M_Z = \frac{\sqrt{\mathrm{g}^2+\mathrm{g}'^2}}{2}\cdot v;\ \ M_A = 0$ for photon.
$e = \mathrm{g}\sin\theta_W, \quad e = \mathrm{g}'\cos\theta_W$

Using $g = \mathrm{g}\sqrt{b_k - 1}$, $\quad g' = \mathrm{g}'\sqrt{b_k - 1}$ we obtain the values of the model parameters g and g' in the primordial action
$g \approx 2.6 \cdot 10^{-3}$ **and** $g' \approx 1.4 \cdot 10^{-3}$
which coincide with their values during slow-roll inflation.

Fermions

We study a model including 3 generations of leptons $l = e, \mu, \tau$

$$L_l = \frac{1-\gamma_5}{2}\begin{pmatrix}\nu^{(l)} \\ l\end{pmatrix}; \qquad l_R = \frac{1+\gamma_5}{2} l; \qquad \nu_R^{(l)} = \frac{1+\gamma_5}{2}\nu^{(l)}$$

and three generations of quarks, and use the following notations for the left $SU(2)$ doublets

$$L_q = \frac{1-\gamma_5}{2}\begin{pmatrix} q \\ K_{qd} d + K_{qs} s + K_{qb} b\end{pmatrix}, \qquad q = u, c, t \qquad (7)$$

and for the right singlets of the up-quarks u, c, t

$$r_q = \frac{1+\gamma_5}{2} q, \qquad q = u, c, t. \qquad (8)$$

Fermions

TMSM provides an unexpected possibility to obtain the observed fermion mass hierarchy in a quite natural way:

Yukawa coupling constant $y^{(ch)}$ is chosen to be universal for three generations of charged leptons,

and similarly, **the Yukawa coupling constant $y^{(up)}$ is chosen to be universal for three generations of up quarks.**

For this we have to choose the minus sign before Υ in the volume measures of the kinetic terms in the primordial action for all fermions:

$(b_l\sqrt{-g} - \Upsilon)$ for charged leptons, $l = e, \mu, \tau$,

and

$(b_q\sqrt{-g} - \Upsilon)$ for up-quarks, $q = u, c, t$.

Fermions

To simplify the first attempt to implement the conceived idea, we completely ignore neutrino mixing and limit ourselves to considering only the electron neutrino ν_e, focusing on a fundamentally new effect that TMSM gives us: explaining the exceptional smallness of the electron neutrino mass.

Like leptons, the quarks Yukawa coupling constants in the primordial action is also chosen to be universal: one for all up-quarks and perhaps another for all down-quarks (with the corresponding modification caused by the Kobayashi-Maskawa mixing matrix). As in the case of leptons, the implementation of such an approach to describing quarks taking into account quark mixing would require too many detailed calculations, which could become an obstacle to demonstrating the main issues of the model under study. Therefore, in this first attempt I ignore the mass generation of down quarks and concentrate only on the description of the up-quarks and their mass generation. That's why with this approach we won't need to use right singlets of the down-quarks.

Fermions

To construct a generally coordinate invariant kinetic terms of the action for fermionic sector we have to use the covariant operator

$\nabla_\mu = D_\mu + \frac{1}{2}\omega_\mu^{ik}\sigma_{ik}$,

where

$D_\mu = \partial_\mu - ig\hat{\mathbf{T}}\mathbf{A}_\mu - i\frac{g'}{2}\hat{Y}B_\mu$,

$\sigma_{ik} = \frac{1}{2}(\gamma_i\gamma_k - \gamma_k\gamma_i)$ and ω_μ^{ik} is the affine spin-connection.

Fermions

The following general coordinate invariant and $SU(2)\times U(1)$ gauge-invariant primordial TMSM action for the lepton sector

$$
\begin{aligned}
S_{(lept)} &= \\
& \int d^4x \sum_{l=e,\mu,\tau}\Bigg[(b_l\sqrt{-g}-\Upsilon)\frac{i}{2}\Big(\overline{L_l}\gamma^\mu\nabla_\mu L_l-(\nabla_\mu\overline{L_l})\gamma^\mu L_l \\
& +\bar{l}_R\gamma^\mu\nabla_\mu l_R-(\nabla_\mu\bar{l}_R)\gamma^\mu l_R\Big) \\
- & \quad (b_l\sqrt{-g}+\Upsilon)\cdot y^{(ch)}\left(\bar{L}_l H l_R+\bar{l}_R H^\dagger L_l\right)\Bigg] \\
+ & \int d^4x(b_e\sqrt{-g}-\Upsilon)\frac{i}{2}\left[\overline{\nu_R}\gamma^\mu\nabla_\mu\nu_R-(\nabla_\mu\overline{\nu_R})\gamma^\mu\nu_R\right] \\
- & \int d^4x(b_\nu\sqrt{-g}+\Upsilon)\cdot y^{(\nu)}\left(\bar{L}_e H_c\nu_R+\overline{\nu_R}H_c^\dagger L_e\right),
\end{aligned}
$$

where $\gamma^\mu=V_k^\mu\gamma^k$, γ^k are Dirac matrices; H_c is the charged conjugated to the Higgs isodoublet H.

Fermions

The general coordinate invariant and $SU(2) \times U(1)$ gauge-invariant primordial TMSM action with the universal Yukawa coupling constant $y^{(up)}$ for the up-quarks

$$
\begin{aligned}
& S_{(quarks)} \qquad (9) \\
= \; & \int d^4x \sum_{q=u,c,t} \Bigg[(b_q\sqrt{-g} - \Upsilon)\frac{i}{2}\Big(\overline{L_q}\gamma^\mu\nabla_\mu L_q - (\nabla_\mu\overline{L_q})\gamma^\mu L_q \\
+ \; & \overline{r_q}\gamma^\mu\nabla_\mu r_q - (\nabla_\mu\overline{r_q})\gamma^\mu r_q\Big) \\
- \; & (b_q\sqrt{-g} + \Upsilon)\cdot y^{(up)}\left(\overline{L_q}H_c r_q + \overline{r_q}H_c^\dagger L_q\right)\Bigg].
\end{aligned}
$$

Of course, as usual in the electroweak model, a standard choice of charges for all fermions is assumed, as well as three color degrees of freedom for the quarks.

Fermions near vacuum

Considering fermions on the cosmological background at the stage near vacuum one can substitute $\zeta = \zeta_v = 1$.
Then, using $(b_l\sqrt{-g} \pm \Upsilon) = \sqrt{-g}(b_l \pm \zeta) = \sqrt{-g}(b_l \pm 1)$ (and similar for neutrino and quarks terms), the action for fermions on the cosmological background at the stage near vacuum takes the form $\int \left(L^{(lept)}_{(on\,back\,near\,vac)} + L^{(quarks)}_{(on\,back\,near\,vac)} \right) \sqrt{-g} d^4x$ where the Lagrangian for leptons is (and similar for quarks):

$$
\begin{aligned}
& L^{(lept)}_{(on\,back\,near\,vac)} = \\
& \sum_{l=e,\mu,\tau} \frac{b_l - 1}{2^{3/2}} \cdot \frac{i}{2} \left[\overline{L_l}\gamma^\mu D_\mu L_l + \overline{l_R}\gamma^\mu D_\mu l_R + h.c. \right] \\
- & \sum_{l=e,\mu,\tau} \frac{b_l + 1}{4} \cdot y^{(ch)} \left(\overline{L_l} H l_R + \overline{l_R} H^\dagger L_l \right) \\
+ & \frac{b_e - 1}{2^{3/2}} \cdot \frac{i}{2} \left[\overline{\nu_R}\gamma^\mu \partial_\mu \nu_R - (\partial_\mu \overline{\nu_R})\gamma^\mu \nu_R \right] \\
- & \frac{b_\nu + 1}{4} y_\nu \left(\overline{L_e} H_c \nu_R + \overline{\nu_R} H_c^\dagger L_e \right)
\end{aligned}
$$

Fermions near vacuum

To reduce this action to canonical form (appropriate for using functional quantization) we have to use the results of the bosonic sector near vacuum, that is to use correspoding redefinitions of the Higgs doublet, gauge bosons and gauge coupling constants, together with a redefinition of the covariant derivative. These manipulations must be supplemented with the following redefinitions of the lepton fields and the Yukawa coupling constants:

$$\frac{\sqrt{b_l - 1}}{2^{3/4}} L_l = L'_l\,, \quad \frac{\sqrt{b_l - 1}}{2^{3/4}} l_R = l'_R, \quad \frac{\sqrt{b_e - 1}}{2^{3/4}} \cdot \nu_R = \nu'_R$$

$$Y^{(l)}_{SM} = \frac{b_l + 1}{(b_l - 1)\sqrt{b_k - 1}} y^{(ch)}, \quad Y^{(\nu)}_{SM} = \frac{b_\nu + 1}{(b_e - 1)\sqrt{b_k - 1}} y_\nu.$$

$l = e, \mu, \tau$

Fermions near vacuum

Then the Lagrangian $L^{(lept)}_{(on\, back\, near\, vac)}$ reduces to the Lagrangian for leptons in the GWS theory:

$$
\begin{aligned}
& L^{(lept}|_{(on\, back\, near\, vac)} \\
= \quad & \sum_{l=e,\mu,\tau} \Bigg[\frac{i}{2} \left(\overline{L'_l} \gamma^\mu \mathcal{D}_\mu L'_l - (\mathcal{D}_\mu \overline{L'_l}) \gamma^\mu L'_l + \overline{l'_R} \gamma^\mu \mathcal{D}_\mu l'_R - (\mathcal{D}_\mu \overline{l'_R}) \gamma^\mu l'_R \right) \\
- \quad & Y^{(l)}_{SM} \left(\overline{L'_l} \mathcal{H} l'_R + \overline{l'_R} \mathcal{H}^\dagger L'_l \right) \Bigg] \\
& + \frac{i}{2} \left[\overline{\nu'_R} \gamma^\mu \frac{\partial \nu'_R}{\partial x^\mu} - \frac{\partial \overline{\nu'_R}}{\partial x^\mu} \gamma^\mu \nu'_R \right] - Y^{(\nu)}_{SM} \left(\overline{L'_e} \mathcal{H}_c \nu'_R + \overline{\nu'_R} \mathcal{H}^\dagger_c L'_e \right) .
\end{aligned}
$$

Fermions near vacuum

Production of fermion masses by the SSB of the gauge symmetry $SU(2) \times U(1) \to U(1)$ at the last stage of cosmological evolution has the same mechanism as in the tree-level GWS theory.

But instead of mass values obtained with specially tuned Yukawa constants as model parameters in the original action, in our TMSM we start from the primordial action with the universal Yukawa coupling constant $y^{(ch)}$ for charged leptons and the universal Yukawa coupling constant $y^{(up)}$ for up-quarks.

As usual in the SM, we obtain expressions for the fermion masses

$$m_l = Y_{SM}^{(l)} \frac{v}{\sqrt{2}} = \frac{b_l + 1}{(b_l - 1)\sqrt{b_k - 1}} y^{(ch)} \frac{v}{\sqrt{2}};$$

$$m_\nu = Y_{SM}^{(\nu)} \frac{v}{\sqrt{2}} = \frac{b_\nu + 1}{(b_e - 1)\sqrt{b_k - 1}} y_\nu \frac{v}{\sqrt{2}},$$

$l = e, \mu, \tau, \quad b_l = b_e, b_\mu, b_\tau$

Fermions near vacuum

Masses of charged leptons and electron neutrino
For charged leptons we choose the universal Yukawa coupling constant

$$y^{(ch)} \approx 10^{-10}.$$

Substituting the values of the masses of charged leptons and $v \approx 246\,GeV$, we obtain the following values of the corresponding parameters

$$b_e \approx 1 + 1.7 \cdot 10^{-2}; \quad b_\mu \approx 1 + 0.8 \cdot 10^{-4}; \quad b_\tau \approx 1 + 4.9 \cdot 10^{-6}.$$

For the electron neutrino (Dirac, active), taking $m_\nu \sim 1eV$ and choosing

$$b_\nu = 1 + \delta_\nu, \quad \delta_\nu \ll 1$$

we obtain $y^{(\nu)} \approx 2 \cdot 10^{-16}$.

Fermions near vacuum

Masses of up-quarks

For up-quarks we choose the universal Yukawa coupling constant

$$y^{(up)} \approx 10^{-9}.$$

Substituting the values of the masses of up-quarks and $v \approx 246\ GeV$, we obtain the following values of the corresponding parameters

$$b_u \approx 1 + 3.7 \cdot 10^{-2}; \quad b_c \approx 1 + 6.7 \cdot 10^{-5}; \quad b_t \approx 1 + 5 \cdot 10^{-7}.$$

Fermions near vacuum

Thus, instead of incorporating Yukawa coupling constants with a spread of several orders of magnitude into the GWS theory, the proposed model realizes the observed fermion mass hierarchy a set of parameters b_i very close to unity. This makes the proposed approach to solving the fermion mass hierarchy problem quite natural.

In this regard, it is also worth paying attention to the fact that in the above calculations, the proximity of the value of another model parameter to 1, namely $b_k \approx 1 + 1.6 \cdot 10^{-5}$, significantly affects the results obtained.

To better understand the complexity of the structure (cosmology+particle physics) of the TMSM, it is important to remember that **this value of b_k was obtained by matching the constraints imposed by the CMB data on the parameters of the inflation model, on the one hand, with experimental data from particle physics at accelerator energies, on the other hand.**

Cosmologically modified copy of the tree-level TMSM on the cosmological background at the stage of the slow-roll inflation (up-copy)

The stage of inflation in the slow-roll regime is realized as $\phi \gtrsim 14.2 M_P$ (corresponding to $\varphi \gtrsim 6 M_P$). We study the SM on the cosmological background at this stage, neglecting relative corrections of the order of $\lesssim 3 \cdot 10^{-2}$. Then one can take the background value of ζ to be $\zeta = 0$.

For bosonic sector, operating with the described precision, we reduce the Lagrangian to the form (ϕ_b is slowly varying background sccalar field; $R_b = -\frac{\lambda}{2\xi^2} M_P^2$ is the background scalar curvature obtained from Einstein eqs.)

$$
\begin{aligned}
& L^{(bos)}_{(on\, back\, \phi_b > 14 M_P)} \\
= \;& \frac{M_P^2}{\xi \phi_b^2} \tilde{g}^{\alpha\beta} \left(D_\alpha H \right)^\dagger D_\beta H - \frac{M_P^4}{2\xi^2 \phi_b^4} \lambda |H|^4 - \frac{M_P^2}{2\xi \phi_b^2} m^2 |H|^2 \\
& - \frac{M_P^2}{\phi_b^2} R_b |H|^2 - \frac{1}{4} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} \left[\mathbf{F}_{\mu\nu} \mathbf{F}_{\alpha\beta} + B_{\mu\nu} B_{\alpha\beta} \right].
\end{aligned}
$$

Bosonic sector of the up-copy

First, we redefine the Higgs field to bring the kinetic term to the canonical form. For this we use the redefinition

$$\tilde{H}(x) = \frac{M_P}{\sqrt{\xi}\phi_b(t)} H(x)$$

Since the background is in a slow-roll inflation stage, it is natural to assume that the background field ϕ_b is a slow dynamical variable compared to $\tilde{H}$, which is subject to quantization at Planck scales and can therefore be considered as a fast variable. In such a picture the following strong inequality must hold

$$\frac{\dot{\phi_b}^2}{\phi_b^2} \ll \frac{|\dot{\tilde{H}}|^2}{|\tilde{H}|^2},$$

Using the estimate $\frac{m^2}{M_P^2} \approx 8 \cdot 10^{-38}$ we see that the second term in $V(\tilde{H}) = \frac{M_P^4}{2\xi^2\phi_b^4}\lambda|H|^4 + \frac{M_P^2}{2\xi\phi_b^2}m^2|H|^2 + \frac{M_P^2}{\phi_b^2}R_b|H|^2$ is negligible compared to the last term. The potential $V(\tilde{H})$ has a minimum at

$$|\tilde{H}_0|^2 = \frac{1}{2}\tilde{v}^2 = -\frac{\xi}{\lambda}R_b = \frac{1}{2\xi}M_P^2 \tag{10}$$

with

$$\tilde{v} = \frac{1}{\sqrt{\xi}}M_P = \sqrt{6}M_P \tag{11}$$

if $\xi = \frac{1}{6}$. This means that, *up to relative corrections of order* $\lesssim 3 \cdot 10^{-2}$*, during the entire slow-roll inflation stage, the potential* $V(\tilde{H})$ of the Higgs field $\tilde{H}$(x) *has a minimum independent of the slowly rolling background field* ϕ_b. However, this minimum cannot be regarded as a stationary vacuum state, since with $\dot{\phi}_b \neq 0$, strictly speaking, it is impossible for the ϕ_b kinetic term to remain exactly zero during slow-roll inflation. The slow change of $\phi_b(t)$ apparently allows one to describe this interesting physical effect as *adiabatic*.

Bosonic sector of the up-copy

After redefinitions the Lagrangian takes the following form

$$L^{(bos)}_{(slow\ roll\ infl)} = \tilde{g}^{\alpha\beta}\left(\tilde{D}_\alpha\tilde{H}\right)^\dagger\tilde{D}_\beta\tilde{H} - V(\tilde{H}) - \frac{1}{4}\tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}\left[\mathbf{F}_{\mu\nu}\mathbf{F}_{\alpha\beta} + B_{\mu\nu}B_{\alpha\beta}\right].$$

$$V(\tilde{H}) = \frac{1}{2}\lambda|\tilde{H}|^4 + \frac{1}{2}m^2|\tilde{H}|^2 + \xi R_b|\tilde{H}|^2.$$

Obviously, this Lagrangian is $SU(2)\times U(1)$ gauge invariant Therefore, quite analogous to how it is done in the GWS theory, one can go to the unitary gauge, where the Higgs isodoublet $\tilde{H}$ reduces to

$$\tilde{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \tilde{v} + \tilde{h}(x) \end{pmatrix}.$$

In accordance with this, we can define a new vacuum state $|\widetilde{0\rangle}$ for which SSB has the form

$$\widetilde{\langle 0}|\tilde{H}|\widetilde{0\rangle} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \tilde{v} \end{pmatrix}, \quad \widetilde{\langle 0}|\tilde{h}|\widetilde{0\rangle} = 0.$$

Bosonic sector of the up-copy

To distinguish this SSB and corresponding vacuum from the SSB and vacuum in the GWS theory, we will use the terms "up-SSB", "up-vacuum" and "up-VEV" for $\tilde{v}$. Similarly, to denote the difference from the Higgs boson h in the GWS theory, for the $\tilde{h}$ boson we will use the term "up-Higgs boson". The potential of the up-Higgs boson $\tilde{h}$ near the up-vacuum reduces to the form

$$V(\tilde{h}) = \frac{\lambda}{8}\left(2\tilde{v}\tilde{h} + \tilde{h}^2\right)^2 - \frac{\lambda}{8\xi^2}M_P^4,$$

from where it follows the expression for the mass squared of the up-Higgs boson $\tilde{h}$

$$m_{\tilde{h}}^2 = \lambda\tilde{v}^2 = 6\lambda M_P^2.$$

Therefore, we get that $m_{\tilde{h}} \approx 2.9 \cdot 10^{13}\ GeV$.

Bosonic sector of the up-copy

The properties of gauge bosons studied at the stage of slow-roll inflation differ significantly from their properties in the GWS theory. Therefore, to avoid confusion, we will use the notations $\tilde{W}$, $\tilde{Z}$ and $\tilde{A}$ for them. Similar to the GWS theory we define

$$\tilde{W}^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(A^1_{\mu} \mp iA^2_{\mu}),$$

$$\tilde{Z}_{\mu} = \sin\theta_W\, B_{\mu} - \cos\theta_W\, B^3_{\mu}, \quad \tilde{A}_{\mu} = \cos\theta_W\, A_{\mu} + \sin\theta_W\, A^3_{\mu},$$

where the Weinberg angle θ_W is is the same as in the GWS theory. Using the values of g and g' we obtain masses of $\tilde{W}$ and $\tilde{Z}$-bosons generated by the up-SSB

$$M_{\tilde{W}} = \frac{g}{2}\cdot\tilde{v} \approx 3.2\cdot 10^{-3} M_P \approx 7.8\cdot 10^{15}\, GeV,$$

$$M_{\tilde{Z}} = \frac{1}{2}\sqrt{g^2 + g'^2}\cdot\tilde{v} \approx 3.6\cdot 10^{-3} M_P \approx 8.8\cdot 10^{15}\, GeV$$

and $M_{\tilde{A}} = 0$ for photon.

Bosonic sector of the up-copy

The relationship between g, g', Weinberg angle θ_W and electric charge is similar to that in the GWS model. But a direct consequence of the values of $g \approx 2.6 \cdot 10^{-3}$, $g' \approx 1.4 \cdot 10^{-3}$ is a “surprising” result:
in the up-copy, the electric charge of the $\tilde{W}$ boson is equal to

$$\tilde{e} = g \sin\theta_W = \mathrm{g}\sqrt{b_k - 1}\,\sin\theta_W = 4 \cdot 10^{-3} e$$

or

$$\tilde{\alpha} = \frac{\tilde{e}^2}{4\pi} = 1.6 \cdot 10^{-5} \alpha$$

Fermionic sector of the up-copy

One can show that at the stage of the slow-roll inflation the TMT effective Yukawa coupling parameters are almost the same for all leptons

$Y^{(ch)}_{eff}|_{(slow\ roll\ infl)} \approx y^{(ch)} \approx 10^{-10}$

and similarly for all up-quarks

$Y^{(up)}_{eff}|_{(slow\ roll\ infl)} \approx y^{(up)} \approx 10^{-9}$

The fermion masses generated by the up-SSB of the $S\tilde{U}(2) \times U(1)$ gauge symmetry are

$$m_{\tilde{e}} \approx m_{\tilde{\mu}} \approx m_{\tilde{\tau}} \approx y^{(ch)} \frac{\tilde{v}}{\sqrt{2}} \approx \sqrt{3} \cdot 10^{-10} M_P,$$

$$m_{\tilde{u}} \approx m_{\tilde{c}} \approx m_{\tilde{t}} \approx y^{(up)} \frac{\tilde{v}}{\sqrt{2}} \approx \sqrt{3} \cdot 10^{-9} M_P,$$

$$m_{\tilde{\nu}} \approx y^{(\nu)} \frac{\tilde{v}}{\sqrt{2}} \approx 2\sqrt{3} \cdot 10^{-16} M_P \sim 5 \cdot 10^2\ GeV.$$

On the vacuum stability in the one-loop approximation during slow-roll inflation

The logarithmic contributions of the gauge fields to the effective potential in the one-loop approximation are of the order

$\sim g^4 \sim g'^4 \sim (10^{-3})^4 \sim 10^{-12}$

The logarithmic (negative) contributions of the fermionic fields to the effective potential in the one-loop approximation are proportional to

$\propto (y^{(ch)})^4 \sim (10^{-10})^4 \sim 10^{-40}$ and

$\propto (y^{(up)})^4 \sim (10^{-9})^4 \sim 10^{-36}$

During the inflation process, one-loop quantum corrections to the effective potential do not violate the stability of the vacuum.

Importantl: all results in TMSM are obtained by

- more fully exploiting the standard mathematical foundations of field theory, starting with spacetime manifolds;
- using only SM fields and $SU(2) \times U(1)$ symmetric action;
- no additional fields or interactions are required.

In this sense, the presented model can be considered as a minimal extension of the SM.
Moreover, unlike other alternative models of gravity, Einstein's general relativity is reproduced exactly.

Thank you for your attention

Appendix to answer possible questions

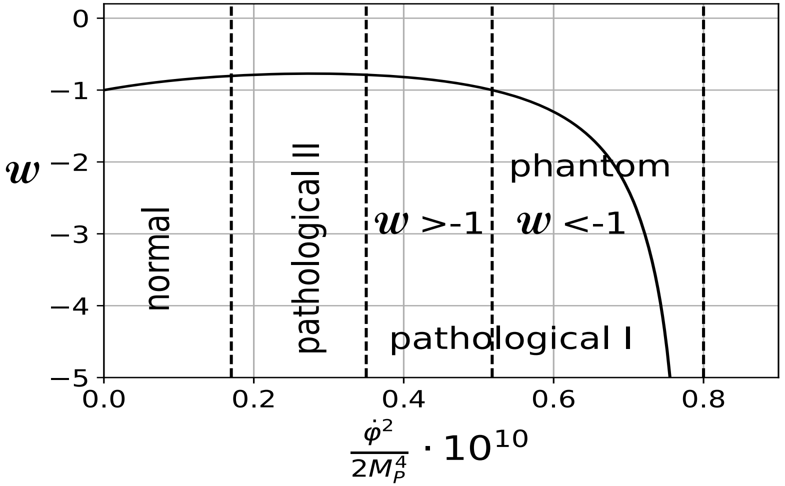

Figure: Plot the $\frac{\dot{\varphi}^2}{2M_P^4}$-dependence of the equation of state $w = \frac{p}{\rho}$ in the region $\varphi > 6M_P$, where the TMT effective potential has a plateau

More about pathological and normal regimes of evolution

It was shown in JCAP 03 (2026) 006 :

1) Pathological regimes of evolution are dynamically isolated from the regime with normal dynamics

2) The initial pathological stage of the evolution of the Universe predicted by TMSM may represent a model describing the unknown physics, the existence of which has been proposed as an interpretation of the Borde-Guth-Vilenkin theorem (Phys.Rev.Lett. 90 (2003) 151301).

3) If evolution begins with normal dynamics, then it inevitably proceeds as slow-roll inflation, meaning that the problem of initial conditions for the onset of inflation does not arise at all.

The effect of the TMT-effective K-essence-type structure on the inflationary parameters ϵ and η and the corresponding flatness conditions

$$\epsilon_k = \frac{M_P^2}{2K_1}\left(\frac{U_{eff}^{(tree)\prime}}{U_{eff}^{(tree)}}\right)^2 \ll 1, \qquad |\eta_k| = \frac{M_P^2}{U_{eff}^{(tree)}\sqrt{K_1}}\left|\left(\frac{U_{eff}^{(tree)\prime}}{\sqrt{K_1}}\right)'\right| \ll 1$$